Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 24 June 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle moves from the point A with position vector $(3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m to the point B with position vector $(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ m under the action of the force $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$ N. Find the work done by the force.

(4)

---

2. A particle P moves in the x-y plane so that its position vector $\mathbf{r}$ metres at time t seconds satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} - 4\mathbf{r} = -3\mathrm{e}^t\mathbf{j}.$$

When $t = 0$, the particle is at the origin and is moving with velocity $(2\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find $\mathbf{r}$ in terms of t.

(10)

---

3. A rocket propels itself by its engine ejecting burnt fuel. Initially the rocket has total mass M, of which a mass kM, $k < 1$, is fuel. The rocket is at rest when its engine is started. The burnt fuel is ejected with constant speed c, relative to the rocket, in a direction opposite to that of the rocket's motion.

Assuming that there are no external forces, find the speed of the rocket when all its fuel has been burnt.

(7)

---

4. Two forces $\mathbf{F}_1 = (3\mathbf{j} + \mathbf{k})$ N and $\mathbf{F}_2 = (4\mathbf{i} + \mathbf{j} - \mathbf{k})$ N act on a rigid body.

The force $\mathbf{F}_1$ acts at the point with position vector $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $(-3\mathbf{i} + 2\mathbf{k})$ m.

The two forces are equivalent to a single force $\mathbf{R}$ acting at the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$.

Find,

(*a*) $\mathbf{R}$,

(2)

(*b*) $\mathbf{G}$.

(4)

A third force $\mathbf{F}_3$ is now added to the system. The force $\mathbf{F}_3$ acts at the point with position vector $(2\mathbf{i} - \mathbf{k})$ m and the three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are equivalent to a couple.

(*c*) Find the magnitude of the couple.

(6)

---

5. A uniform rod PQ, of mass m and length $2a$, is made to rotate in a vertical plane with constant angular speed $\sqrt{\left(\frac{g}{a}\right)}$ about a fixed smooth horizontal axis through the end P of the rod.

Show that, when the rod is inclined at an angle θ to the downward vertical, the magnitude of the force exerted on the axis by the rod is $2mg|\cos(\frac{1}{2}\theta)|$.

(8)

---

6. A uniform rod AB of mass $4m$ is free to rotate in a vertical plane about a fixed smooth horizontal axis, L, through A. The rod is hanging vertically at rest when it is struck at its end B by a particle of mass m. The particle is moving with speed u, in a direction which is horizontal and perpendicular to L, and after striking the rod it rebounds in the opposite direction with speed v. The coefficient of restitution between the particle and the rod is 1.

Show that $u = 7v$.

(7)

---

7. Prove, using integration, that the moment of inertia of a uniform solid right circular cone, of mass M and base radius a, about its axis is $\frac{3}{10}Ma^2$.

[You may assume, without proof, that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to its plane is $\frac{1}{2}mr^2$.]

(10)

---

8. A pendulum consists of a uniform rod PQ, of mass $3m$ and length $2a$, which is rigidly fixed at its end Q to the centre of a uniform circular disc of mass m and radius a. The rod is perpendicular to the plane of the disc. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through the end P of the rod and is perpendicular to the rod.

(*a*) Show that the moment of inertia of the pendulum about L is $\frac{33}{4}ma^2$.

(5)

The pendulum is released from rest in the position where PQ makes an angle α with the downward vertical. At time t, PQ makes an angle θ with the downward vertical.

(*b*) Show that the angular speed, $\dot{\theta}$, of the pendulum satisfies

$$\dot{\theta}^2 = \frac{40g(\cos\theta - \cos\alpha)}{33a}.$$

(4)

(*c*) Hence, or otherwise, find the angular acceleration of the pendulum.

(3)

Given that $\alpha = \frac{\pi}{20}$ and that PQ has length $\frac{8}{33}$ m,

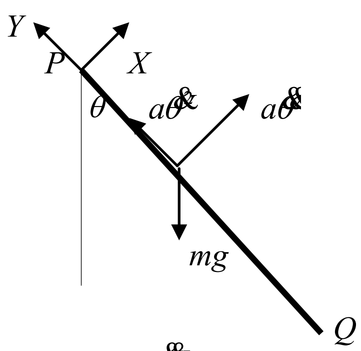
(*d*) find, to 3 significant figures, an approximate value for the angular speed of the pendulum 0.2 s after it has been released from rest.

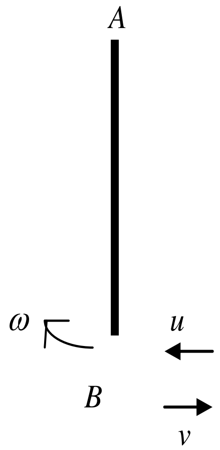
(5)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\mathbf{AB} = (\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}) - (3\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = (-2\mathbf{i} - \mathbf{j} - 7\mathbf{k})$ $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k}).(-2\mathbf{i} - \mathbf{j} - 7\mathbf{k}) = -4 + 3 + 7 = 6$ J	M1 A1 M1 A1 **4**
2.	$m^2 - 4 = 0 \Rightarrow m = 2 or -2$ CF is $\mathbf{r} = \mathbf{A}e^{2t} + \mathbf{B}e^{-2t}$ PI try $\mathbf{r} = \mathbf{C}e^t$ $\dot{\mathbf{r}} = \mathbf{C}e^t$ $\ddot{\mathbf{r}} = \mathbf{C}e^t$ $\mathbf{C}e^t - 4\mathbf{C}e^t = -3e^t\mathbf{j}$ $\mathbf{C} = \mathbf{j}$ GS is $\mathbf{r} = \mathbf{A}e^{2t} + \mathbf{B}e^{-2t} + \mathbf{j}e^t$ $\mathbf{v} = 2\mathbf{A}e^{2t} - 2\mathbf{B}e^{-2t} + \mathbf{j}e^t$ $t = 0, \mathbf{r} = \mathbf{0}, \mathbf{v} = 2\mathbf{i} + \mathbf{j}$ $\mathbf{0} = \mathbf{A} + \mathbf{B} + \mathbf{j}$ $2\mathbf{i} + \mathbf{j} = 2\mathbf{A} - 2\mathbf{B} + \mathbf{j}$ $\mathbf{i} = \mathbf{A} - \mathbf{B}$ $\mathbf{A} = \frac{1}{2}(\mathbf{i} - \mathbf{j}); \mathbf{B} = -\frac{1}{2}(\mathbf{i} + \mathbf{j})$ $\mathbf{r} = \frac{1}{2}(\mathbf{i} - \mathbf{j})e^{2t} - \frac{1}{2}(\mathbf{i} + \mathbf{j})e^{-2t} + \mathbf{j}e^t$	M1 A1 B1 M1 A1 A1 M1 M1 A1 A1 **10**
3.	$(m + \delta m)(v + \delta v) + (-\delta m)(v - c) = mv$ $m\delta v + c\delta m = 0$ $\int_0^V \mathrm{d}v = -c\int_M^{M(1-k)} \frac{\mathrm{d}m}{m}$ $V = c[\ln m]_{M(1-k)}^{M}$ $V = c\ln\left(\frac{1}{1-k}\right)$	M1A2 M1A1 A1 A1 **7**

Question Number	Scheme	Marks
4.		
(a)	$\mathbf{R} = (3\mathbf{j}+\mathbf{k}) + (4\mathbf{i}+\mathbf{j}-\mathbf{k})$	M1
	$=(4\mathbf{i}+4\mathbf{j})$ (N)	A1
		(2)
(b)	$(\mathbf{i}+2\mathbf{j}+\mathbf{k})\times(4\mathbf{i}+4\mathbf{j})+\mathbf{G} = (2\mathbf{i}-\mathbf{j}+3\mathbf{k})\times(3\mathbf{j}+\mathbf{k})+(-3\mathbf{i}+2\mathbf{k})\times(4\mathbf{i}+\mathbf{j}-\mathbf{k})$	M1
	$(-4\mathbf{i}+4\mathbf{j}-4\mathbf{k})+\mathbf{G} = (-10\mathbf{i}-2\mathbf{j}+6\mathbf{k})+(-2\mathbf{i}+5\mathbf{j}-3\mathbf{k})$	A2
	$\mathbf{G} = (-8\mathbf{i}-\mathbf{j}+7\mathbf{k})$ (N m)	A1
		(4)
(c)	$\mathbf{F}_3 = -\mathbf{R} = (-4\mathbf{i}-4\mathbf{j})$	B1
	$\mathbf{G} = (2\mathbf{i}-\mathbf{k})\times(-4\mathbf{i}-4\mathbf{j})+(-12\mathbf{i}+3\mathbf{j}+3\mathbf{k})$	M1 A1
	$=(-16\mathbf{i}+7\mathbf{j}-5\mathbf{k})$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{(-16)^2+7^2+(-5)^2}$	M1
	$=\sqrt{330}$ (N m)	A1 (6)
		12
5.		
	$\ddot{\theta} = 0$	B1
	$X - mg\sin\theta = ma\ddot{\theta} (=0)$	M1 A1
	$X = mg\sin\theta$	
	$Y - mg\cos\theta = ma\dot{\theta}^2 = ma\frac{g}{a} = mg$	M1 A1
	$Y = mg(1+\cos\theta)$	
	$R = mg\sqrt{(1+\cos\theta)^2+\sin^2\theta}$	M1
	$= mg\sqrt{2(1+\cos\theta)}$	
	$= mg\sqrt{2.2\cos^2(\frac{1}{2}\theta)}$	DM1
	$= 2mg\left\lvert\cos(\frac{1}{2}\theta)\right\rvert$ *	A1
		8

Question Number	Scheme	Marks
6.		
	$I_A = \frac{1}{3}4ml^2$	B1
	CAM: $mul = \frac{1}{3}4ml^2\omega - mvl$	M1 A1
	$3u = 4l\omega - 3v$	
	NIL: $u = \omega l + v$	M1 A1
	eliminating ωl	DM1
	$u = 7v*$	A1
		7
7.	$r_x = \frac{rx}{h}$	M1A1
	$\delta m = \pi r_x^2 \delta x.\rho$	M1
	$= \pi(\frac{rx}{h})^2 \delta x.\frac{3M}{\pi r^2 h}$	
	$= \frac{3M}{h^3}x^2\delta x$	A1
	$\delta I = \frac{1}{2}\delta m\ r_x^2$	M1A1
	$= \frac{1}{2}\frac{3M}{h^3}x^2\delta x(\frac{rx}{h})^2$	
	$= \frac{3Mr^2}{2h^5}x^4\delta x$	A1 (DM1)
	$I = \frac{3Mr^2}{2h^5}\int_0^h x^4\ dx$	M1
	$= \frac{3Mr^2}{2h^5}\left[\frac{x^5}{5}\right]_0^h$	A1
	$= \frac{3Mr^2}{10}$	A1
		10

Question Number	Scheme	Marks
8. **(a)**	$I_{DISC} = \frac{ma^2}{4} + m(2a)^2 = \frac{17ma^2}{4}$	M1A1
	$I_{ROD} = \frac{3m(2a)^2}{3} = 4ma^2$	B1
	$I_{PENDULUM} = \frac{17ma^2}{4} + 4ma^2 = \frac{33ma^2}{4}$	M1 A1
		(5)
(b)	$3mga(\cos\theta - \cos\alpha) + mg.2a(\cos\theta - \cos\alpha) = \frac{1}{2}\frac{33ma^2}{4}\dot{\theta}^2$	M1A2
	$\frac{40g(\cos\theta - \cos\alpha)}{33a} = \dot{\theta}^2$ *	A1
		(4)
(c)	$2\dot{\theta}\ddot{\theta} = -\frac{40g}{33a}\sin\theta.\dot{\theta}$	M1A1
	$\ddot{\theta} = -\frac{20g}{33a}\sin\theta$	A1
		(3)
(d)	For small θ, $\ddot{\theta} = -\frac{20g}{33a}\theta$ i.e. SHM	M1
	$\omega = \sqrt{\frac{20g}{33a}} = \sqrt{\frac{20g}{33\text{x}\frac{4}{33}}} = 7$	A1
	$\theta = \alpha\cos\omega t$	M1
	$\dot{\theta} = -\alpha\omega\sin\omega t$	M1
	$= -7\frac{\pi}{20}\sin 1.4$	
	$\lvert\dot{\theta}\rvert = 1.08$ rad s^{-1} (3SF)	A1
		(5) **17**